PB13

ON SIMULATING A MEDIUM WITH THE PROPERTY OF THE IDEAL MIRROR FOR THE LIGHT AND SPIN 1/2 PARTICLES

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Maxwell equations formulated on the background of the Lobachevsky geometry in quasi-cartesian coordinates (*x*, *y*, *z*)

$$dS^2 = dt^2 - e^{-2z}(dx^2 + dy^2) - dz^2 ,$$

can be understood as the Maxwell equations in Minkowski space but in a special effective medium [1–2]:

$$\varepsilon^{ik}(x) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-2z} \end{vmatrix}, \quad (\mu^{-1})^{ik}(x) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{2z} \end{vmatrix},$$

which provides the constitutive equations $D^i = \varepsilon_0 \varepsilon^{ik} E_k$, $B_i = \mu_0 \mu^{ik} H^k$, $\varepsilon^{ik}(x) = \mu^{ik}(x)$. Thus, this effective medium is inhomogeneous along the axis *z*. The Maxwell's equations have been examined within three-dimensional complex formalism of Majorana-Oppenheimer [3]

$$\left(-i\frac{\partial}{\partial t} + \alpha^{(1)} e^z \frac{\partial}{\partial x} + \alpha^{(2)} e^z \frac{\partial}{\partial y} + \alpha^{(3)} \frac{\partial}{\partial z} - \alpha^{(1)} s_2 + \alpha^{(2)} s_1 \right) \begin{vmatrix} 0 \\ \mathbf{E} + i\mathbf{B} \end{vmatrix} = 0 ;$$

explicit form of the matrices is given in [3]. After separation of the variables through the substitution

$$\begin{vmatrix} 0 \\ \mathbf{E} + i\mathbf{B} \end{vmatrix} = e^{-i\omega t} e^{iax} e^{iby} \begin{vmatrix} 0 \\ \mathbf{f}(z) \end{vmatrix},$$

we get three equations

$$f_1 = e^z F_1(z), \; f_2 = e^z F_2(z),$$

$$f_3 = \frac{-ib}{\omega} e^{2z} F_1 + \frac{ia}{\omega} e^{2z} F_2, \; e^z = \sqrt{\omega} Z,$$

$$Z\left(\frac{d}{dZ} + abZ \right) F_2 = +(b^2 Z^2 - \omega) F_1,$$

$$Z\left(\frac{d}{dZ} - abZ \right) F_1 = -(a^2 Z^2 - \omega) F_2 .$$

With the help of linear transformation

$$F_1 = +\frac{b}{\sqrt{a^2 + b^2}} G_1 + \frac{a}{\sqrt{a^2 + b^2}} G_2 ,$$

$$F_2 = -\frac{a}{\sqrt{a^2 + b^2}} G_1 + \frac{b}{\sqrt{a^2 + b^2}} G_2$$

the problem is reduced to the differential equation with simple singular points (0 and ∞):

$$Z\frac{d}{dZ} G_1 = \omega G_2 , \; Z\frac{d}{dZ} G_2 = \left[Z^2 (a^2 + b^2) - \omega \right] G_1 .$$

Further, the problem is described by Schrödinger like one-dimensional equation with an effective potential

$$\left(\frac{d^2}{dz^2} + \omega^2 - (a^2 + b^2)e^{2z}\right)G_1 = 0\,,$$

which is illustrated by the Fig. 1.

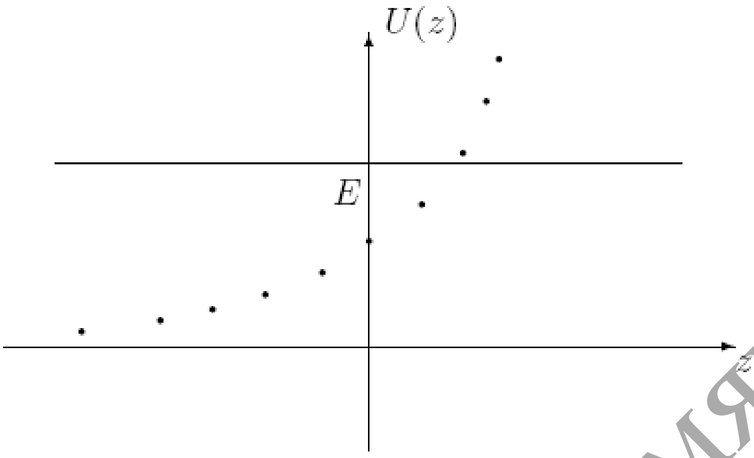

Fig. 1. Effective potential function $U(z)$

In the context of quantum mechanics, this equation describes the motion of a particle in a potential field, gradually increasing to infinity in the z coordinate striving to infinity; particle is reflected from the barrier and does not penetrate him. A similar situation occurs also in electrodynamics.

Thus, Lobachevsky geometry simulates an effective perfect mirror, distributed in space and oriented perpendicularly to the z-axis. The field penetration depth z_0 into the «medium-mirror» is given by the relation

$$z_0 = \rho \ln \frac{\omega}{c\sqrt{k_1^2 + k_2^2}}\,;$$

it is defined by parameters of solutions and the curvature radius ρ of the Lobachevsky space.

Similar analysis has been performed for a spin S particle. Influence of the geometry on the particles with spin 1/2 (nonrelativistic electron or neutron described by a generalized Pauli equation on the background of the non-Euclidean geometry) is the same: «medium» acts on the fermions as the perfect mirror, depth of penetration of particles with spin increases with energy and decreases with increasing the curvature of space.

REFERENCES

[1] V.M. Red’kov, N.G. Tokarevskaya, E.M. Ovsiyuk, George J. Spix. NPCS, **12** (3), 232–250 (2009).

[2] E.M. Ovsiyuk, O.V. Veko, V.M. Red’kov. NPCS, **16** (4), 331–344 (2013).

[3] E.M. Ovsiyuk, V.V. Kisel, V.M. Red’kov. Maxwell Electrodynamics and Boson Fields in Spaces of Constant Curvature, New York, Nova Science Publishers Inc. (2014).