**M. NEAGU**[1], **E.M. OVSIYUK**[2]
[1]Transilvania University of Braşov (Braşov, Romania)
[2]Mozyr State Pedagogical University named after I.P. Shamyakin (Mozyr, Belarus)

# RABINOVICH-FABRIKANT DYNAMICAL SYSTEM AND LAGRANGE-HAMILTON GEOMETRIES

In this paper, via the least squares variational method, we develop the Lagrange-Hamilton geometries (in the sense of nonlinear connections, d-torsions and Lagrangian Yang-Mills electromagnetic-like energy) induced by the well known Rabinovich-Fabrikant dynamical system that exhibits a chaotic behaviour.

1 Lagrange-Hamilton geometries produced by a given dynamical system

Let $M$ be a $n$-dimensional smooth manifold, whose coordinates are $\left(x^i\right)_{i=\overline{1,n}}$. Let $TM$ (respectively $T^*M$) be the tangent (respectively cotangent) bundle, whose coordinates are $\left(x^i, y^i\right)_{i=\overline{1,n}}$ (respectively $\left(x^i, p_i\right)_{i=\overline{1,n}}$).

Let us consider a vector field $X = \left(X^i(x)\right)_{i=\overline{1,n}}$ on $M$, which produces the dynamical system

$$\frac{dx^i}{dt} = X^i(x(t)), \quad i = \overline{1,n}. \tag{1}$$

Because the solutions of class $C^2$ of the dynamical system (1) are the global minimum points for the *least squares Lagrangian* $L : TM \to \mathrm{R}$, given by (see the book [1])

$$L(x,y) = \delta_{ij}\left(y^i - X^i(x)\right)\left(y^j - X^j(x)\right), \tag{2}$$

it follows that, via its Euler-Lagrange equations, we can construct an entire collection of nonzero Lagrangian geometrical objects (such as nonlinear connection, d-torsions and Yang-Mills electromagnetic-like energy) that characterize the initial dynamical system (1).

Also, if we construct the *least squares Hamiltonian* $H : T^*M \to \mathrm{R}$, associated with the Lagrangian (2), which is defined by (see [3])

$$H(x,p) = \frac{\delta^{ij}}{4} p_i p_j + X^k(x) p_k, \tag{3}$$

where $p_r = \partial L / \partial y^r$ and $H = p_r y^r - L$, we can build a collection of nonzero Hamiltonian geometrical objects (such as nonlinear connection and d-torsions), which also characterize the system (1).

It is important to note that the above Lagrange-Hamilton geometries produced by the Lagrangian (2) and Hamiltonian (3) are exposed in details in the monographs [1] and [3]. These are achieved via the nonzero geometrical objects:

1. $N = \left(N^i_j\right)_{i,j=\overline{1,n}} = -\frac{1}{2}\left[J(X) - {}^T J(X)\right]$ – the *Lagrangian nonlinear connection*;

2. $R_k = \left(R^i_{jk}\right)_{i,j=\overline{1,n}} = \frac{\partial N}{\partial x^k}, \ \forall \ k = \overline{1,n},$ – the *Lagrangian d-torsions*;

3. $\mathrm{EYM}\,(x) = \frac{1}{2} \cdot Trace\left[F \cdot {}^T F\right]$, where $F = -N$, – the *Yang-Mills electromagnetic-like energy*;

4. $\mathbf{N} = \left(N_{ij}\right)_{i,j=\overline{1,n}} = J(X) + {}^{T}J(X)$ – the *Hamiltonian nonlinear connection*;

5. $\mathbf{R}_k = \left(R_{kij}\right)_{i,j=\overline{1,n}} = \frac{\partial}{\partial x^k}\left[J(X) - {}^{T}J(X)\right] = -2R_k, \quad \forall \quad k = \overline{1,n},$ – the *Hamiltonian d-torsions*, where $J(X) = \left(\partial X^i / \partial x^j\right)_{i,j=\overline{1,n}}$ is the Jacobian matrix of $X$.

2 Lagrange-Hamilton geometries for Rabinovich-Fabrikant dynamical system

If we take the particular 3-dimensional manifold $M = \mathbb{R}^3$, whose coordinates are $\left(x^1 = x, x^2 = y, x^3 = z\right)$, and we consider the vector field $X = \left(X^i(x,y,z)\right)_{i=\overline{1,3}}$, where

$$X^1(x,y,z) = y\left(z - 1 + x^2\right) + \gamma x, \quad X^2(x,y,z) = x\left(3z + 1 - x^2\right) + \gamma y,$$

$$X^3(x,y,z) = -2z\left(\nu + xy\right), \quad \gamma, \nu > 0,$$

then we find the well known Rabinovich-Fabrikant (RF) dynamical system initially written in 1979 (see [4]).

**Remark 1** *The RF dynamical system is used in Physics and Engineering because it allows the unexpected and potential responses to perturbations in a structure like a bridge or aircraft wing [2].*

The Jacobian matrix $J = J(X)$ of the vector field $X(x,y,z)$ is expressed by

$$J = \begin{pmatrix} 2xy + \gamma & z - 1 + x^2 & y \\ 3z + 1 - 3x^2 & \gamma & 3x \\ -2yz & -2xz & -2\left(\nu + xy\right) \end{pmatrix},$$

and, consequently, we find the Lagrange-Hamilton geometrical objects that characterize the RF dynamical system:

1. the Lagrangian nonlinear connection matrix:

$$N = -\frac{1}{2}\left[J - {}^{T}J\right] = \begin{pmatrix} 0 & z + 1 - 2x^2 & -y/2 - yz \\ -z - 1 + 2x^2 & 0 & -3x/2 - xz \\ y/2 + yz & 3x/2 + xz & 0 \end{pmatrix};$$

2. the Lagrangian d-torsion matrices:

$$R_1 = \frac{\partial N}{\partial x} = \begin{pmatrix} 0 & -4x & 0 \\ 4x & 0 & -3/2 - z \\ 0 & 3/2 + z & 0 \end{pmatrix},$$

$$R_2 = \frac{\partial N}{\partial y} = \begin{pmatrix} 0 & 0 & -1/2 - z \\ 0 & 0 & 0 \\ 1/2 + z & 0 & 0 \end{pmatrix},$$

$$R_3 = \frac{\partial N}{\partial z} = \begin{pmatrix} 0 & 1 & -y \\ -1 & 0 & -x \\ y & x & 0 \end{pmatrix};$$

3. the Lagrangian Yang-Mills electromagnetic-like energy:

$$\text{EYM } (x, y, z) = \left(z + 1 - 2x^2\right)^2 + \left(\frac{y}{2} + yz\right)^2 + \left(\frac{3x}{2} + xz\right)^2;$$

4. the Hamiltonian nonlinear connection matrix:

$$N = J + {}^T J = \begin{pmatrix} 4xy + 2\gamma & 4z - 2x^2 & y - 2yz \\ 4z - 2x^2 & 2\gamma & 3x - 2xz \\ y - 2yz & 3x - 2xz & -4\left(\nu + xy\right) \end{pmatrix};$$

5. the Hamiltonian d-torsion matrices are $\mathbf{R}_k = -2R_k, \;\; \forall \;\; k = \overline{1,3}$.

**Open problem.** The surfaces of constant level of the Lagrangian Yang-Mills electromagnetic-like energy produced by the RF dynamical system could have important connotations for the physical phenomena taken in study. For such a reason, it is an open problem to find the physical information contained in the shape of the surfaces of constant level

$$\Sigma_C : \left(z + 1 - 2x^2\right)^2 + \left(\frac{y}{2} + yz\right)^2 + \left(\frac{3x}{2} + xz\right)^2 = C > 0.$$

In this direction, we believe that the computer drawn graphics of these surfaces are important for the study of the physical phenomena involved in the RF dynamical system.

REFERENCES

1. Balan, V. Jet Single-Time Lagrange Geometry and Its Applications / V. Balan, M. Neagu. – Hoboken, New Jersey : John Wiley & Sons, Inc., 2011.
2. Gupta, M.K. Rabinovich-Fabrikant system in view point of KCC theory in Finsler geometry / M.K. Gupta, C.K. Yadav // Journal of Interdisciplinary Mathematics. – 2019. – Vol. 22, iss. 3. – P. 219–241.
3. Neagu, M. Dual Jet Geometrization for Time-Dependent Hamiltonians and Applications / M. Neagu, A. Oana // Synthesis Lectures on Mathematics & Statistics. – Cham, Switzerland : Springer, 2022.
4. Rabinovich, M.I. Stochastic self-modulation of waves in nonequilibrium media / M.I. Rabinovich, A.L. Fabrikant // Zh. Eksp. Teor. Fiz. – 1979. – Vol. 77. – P. 617–629.

**A.I. SERY**
Brest State University named after A.S. Pushkin (Brest, Belarus)

## ON DIFFERENT CONDITIONS OF THE EXISTENCE OF ELECTRON-PROTON-NEUTRON MATTER

Electron-proton-neutron ($enp$-) matter is one of the objects of research of modern astrophysics of superdense matter [1, p. 167–186; 2, p. 270–272; 3, p. 506–512]. Since the conditions of its existence can be different (which leads to a difference in a number of characteristics), a comparative analysis of various models in which the appearance of $enp$-phase is possible is of interest.

The results of such a comparative analysis are presented below in the form of a table.