Секция 2

Теоретическая физика

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COMPLETELY AND PARTIALLY POLARIZED LIGHT, JONES 4-SPINOR, STOKES VECTOR AND TENSOR

In the present paper, starting with the Lorentz group theory, we introduce the concept of 4-dimensional Jones spinor, first for a completely polarized light. To such Jones bispinor, there correspond isotropic Stokes 4-vector, and antisymmetric Stokes tensor, the last is equivalent to isotropic complex 3-vector. This approach is extended to the partially polarized light as well. We introduce corresponding 4-spinor of Jones type and 4-vector and antisymmetric tensor of Stokes types.

Keywords: polarized light, Jones 4-spinor, Stokes 4-vector, antisymmetric Stokes tensor.

Let us start with the well-known relations between a 2-rank bispinor and simplest tensors. The bispinor of second rank $U = \Psi \otimes \Psi$ can be decomposed into the scalar Φ, vector Φ_b, pseudoscalar $\tilde{\Phi}$, pseudovector $\tilde{\Phi}_b$, and skew-symmetric tensor Φ_{ab}, as follows

$$U = \Psi \otimes \Psi = (-i\Phi + \gamma^b S_b + i\sigma^{ab} S_{ab} + \gamma^5 \tilde{S} + i\gamma^b \gamma^5 \tilde{S}_b) E^{-1}, E = \begin{vmatrix} i\sigma^2 & 0 \\ 0 & -i\sigma^2 \end{vmatrix}; \qquad (1)$$

we shall refer all the subsequent considerations to the spinor basis

$$U = \begin{vmatrix} \xi & \Delta \\ H & \eta \end{vmatrix}, \quad \gamma^a = \begin{vmatrix} 0 & \bar{\sigma}^a \\ \sigma^a & 0 \end{vmatrix}, \quad \gamma^5 = \begin{vmatrix} -I & 0 \\ 0 & +I \end{vmatrix}, \quad \sigma^{ab} = \begin{vmatrix} \bar{\sigma}^a \sigma^b - \bar{\sigma}^b \sigma^a & 0 \\ 0 & \sigma^a \bar{\sigma}^b - \sigma^b \bar{\sigma}^a \end{vmatrix}.$$

The inverses of the relations (1) have the form

$$S_a = \frac{1}{4} Sp[E\gamma_a U], \tilde{S}_a = \frac{1}{4i} Sp[E\gamma^5 \gamma_a U],$$

$$S = \frac{i}{4} Sp[EU], \tilde{S} = \frac{1}{4} Sp[E\gamma^5 U], S^{mn} = -\frac{1}{2i} Sp[E\sigma^{mn} U].$$

In order that the vector and the tensor be both real, one should impose additional restrictions; the first constraint of Majorana type may be taken in the form

$$\Psi = \begin{vmatrix} \eta \\ \xi \end{vmatrix}, \eta = -i\,\sigma^2\, \xi^* \qquad \Rightarrow \qquad \eta_1 = -\xi^{2*}, \ \eta_2 = +\xi^{1*},$$

which implies (note that $\Phi_0 > 0$.):

$$\Psi = \begin{vmatrix} \xi \\ \eta = -i\,\sigma^2\,\xi^* \end{vmatrix}, \quad \Psi \otimes \Psi \;\Rightarrow\; S_a \neq 0,\; S_{mn} \neq 0\,,$$

$$S_0 = (\xi^1\,\xi^{1*} + \xi^2\,\xi^{2*}) > 0\,, \qquad S_3 = -(\xi^1\,\xi^{1*} - \xi^2\,\xi^{2*})\,,$$

$$S_1 = -(\xi^1\,\xi^{2*} + \xi^2\,\xi^{1*})\,, \qquad S_2 = -i\,(\xi^1\,\xi^{2*} - \xi^2\,\xi^{1*})\,,$$

$$a_1 = S^{01} = \frac{i}{4}[\,(\xi^1\,\xi^1 - \xi^2\,\xi^2) + (\xi^{2*}\,\xi^{2*} - \xi^{1*}\,\xi^{1*})\,]\,,$$

$$b_1 = S^{23} = \frac{1}{4}[\,(\xi^1\,\xi^1 - \xi^2\,\xi^2) - (\xi^{2*}\,\xi^{2*} - \xi^{1*}\,\xi^{1*})\,]\,,$$

$$a_2 = S^{02} = -\frac{1}{4}[\,(\xi^1\,\xi^1 + \xi^2\,\xi^2) + (\xi^{2*}\,\xi^{2*} + \xi^{1*}\,\xi^{1*})\,]\,,$$

$$b_2 = S^{31} = -\frac{1}{4i}[\,(\xi^1\,\xi^1 + \xi^2\,\xi^2) - (\xi^{2*}\,\xi^{2*} + \xi^{1*}\,\xi^{1*})\,]\,,$$

$$a_3 = S^{03} = -\frac{i}{2}(\,\xi^1\,\xi^2 - \xi^{2*}\,\xi^{1*})\,, \quad b_3 = S^{12} = -\frac{1}{2}(\,\xi^1\,\xi^2 + \xi^{2*}\,\xi^{1*})\,.$$

The last case seems to be appropriate to describe the Stokes 4-vector and to determine the Stokes 2-rank tensor (existence of the Stokes-type 2-rank tensor seems to be unexpected). Additionally, let us calculate the main invariant for the 4-vector, which turns to be equal to zero

$$S_0 S_0 - S_j S_j = \frac{1}{4}[\,(\xi^1\,\xi^{1*} + \xi^2\,\xi^{2*})^2 - (\xi^1\,\xi^{1*} - \xi^2\,\xi^{2*})^2$$

$$-(\xi^1\,\xi^{2*} + \xi^2\,\xi^{1*})^2 + (\xi^1\,\xi^{2*} - \xi^2\,\xi^{1*})^2\,] = (\xi^1\xi^{1*})\,(\xi^2\xi^{2*}) - (\xi^1\xi^{2*})\,(\xi^2\xi^{1*}) \equiv 0\,;$$

so the quantity S_a may be considered as a Stokes 4-vector for completely polarized light.

In turn, the 4-tensor S_{mn}, being constructed from the Jones bi-spinor Ψ, will be called a Stokes 2-rank tensor [1], [2]. Let us calculate the two invariants for S_{mn}:

$$I_1 = -\frac{1}{2}S^{mn}S_{mn} \equiv 0\,, \quad I_2 = \frac{1}{4}\epsilon_{abmn}S^{ab}S^{mn} = a \cdot b \equiv 0\,.$$

Instead of considering the Stokes 4-tensor S_{ab} one may introduce a complex Stokes 3-vector with the following components

$$s_1 = a^1 + ib^1 = S^{01} + iS^{23} = \frac{i}{2}(\xi^1\,\xi^1 - \xi^2\,\xi^2)\,,$$

$$s_2 = a^2 + ib^2 = S^{02} + iS^{31} = -\frac{1}{2}(\xi^1\,\xi^1 + \xi^2\,\xi^2)\,,$$

$$s_3 = a^3 + ib^3 = S^{03} + iS^{12} = -i\,\xi^1\,\xi^2\,;$$

whence it follows

$$s_1 + is_2 = -i\,\xi^2\xi^2\,, \qquad s_1 - is_2 = +i\,\xi^1\xi^1\,, \qquad s_3 = -i\,\xi^1\xi^2\,.$$

The quantity $\vec{s}$ transforms as a vector under the complex rotation group $SO(3,C)$, recall that this group is isomorphic to the Lorentz group. This allows to employ additionally to the Jones spinor and Stokes vector formalisms one other technique, based on the use of complex 3-vector under the complex rotation group $SO(3,C)$: $\vec{s} = \vec{a} + i\vec{b}, \vec{s}^{\,2} \equiv 0$.

Let us define the Jones 4-spinor for partially polarized light. To this end, let us examine other possibility to construct tensors from 4-spinors:

$$U = \Psi \otimes \Psi^c = \begin{vmatrix} \xi^1 \\ \xi^2 \\ \eta_{\dot{1}} \\ \eta_{\dot{2}} \end{vmatrix} \otimes \begin{vmatrix} +\eta_{\dot{2}}^* \\ -\eta_{\dot{1}}^* \\ -\xi^{2*} \\ +\xi^{1*} \end{vmatrix} = \begin{vmatrix} A \\ B \\ C \\ D \end{vmatrix} \otimes \begin{vmatrix} +D^* \\ -C^* \\ -B^* \\ +A^* \end{vmatrix} = \begin{vmatrix} AD^* & -AC^* & -AB^* & +AA^* \\ BD^* & -BC^* & -BB^* & +BA^* \\ CD^* & -CC^* & -CB^* & +CA^* \\ DD^* & -DC^* & -DB^* & +DA^* \end{vmatrix}.$$

We readily derive expression for corresponding tensors:

the scalar and the pseudo-scalar

$$S = -\frac{1}{4i}(AC^* + BD^* + CA^* + DB^*), \quad \tilde{S} = -\frac{1}{4}(AC^* + BD^* - CA^* - DB^*);$$

the 4-vector (real-valued)

$$S^0 = \frac{1}{4}(AA^* + BB^* + DD^* + CC^*), S^3 = \frac{1}{4}(AA^* - BB^* + DD^* - CC^*),$$

$$S^1 = \frac{1}{4}(AB^* + BA^* - CD^* - DC^*), S^2 = -\frac{i}{4}(-AB^* + BA^* + CD^* - DC^*);$$

the pseudo-vector (imaginary)

$$\tilde{S}^0 = \frac{1}{4i}(AA^* + BB^* - DD^* - CC^*), \tilde{S}^3 = \frac{1}{4i}(AA^* - BB^* - DD^* + CC^*),$$

$$\tilde{S}^1 = \frac{1}{4i}(AB^* + BA^* + CD^* + DC^*), \tilde{S}^2 = -\frac{1}{4}(-AB^* + BA^* - CD^* + DC^*),$$

the skew-symmetric tensor (real)

$$S^{01} = \frac{i}{4}(AD^* + BC^* - CB^* - DA^*), S^{23} = \frac{1}{4}(AD^* + BC^* + CB^* + DA^*),$$

$$S^{02} = -\frac{1}{4}(AD^* - BC^* - CB^* + DA^*), S^{31} = \frac{i}{4}(AD^* - BC^* + CB^* - DA^*),$$

$$S^{03} = -\frac{i}{4}(-AC^* + BD^* + CA^* - DB^*), S^{12} = -\frac{1}{4}(-AC^* + BD^* - CA^* + DB^*),$$

$$s_1 = \frac{i}{2}(AD^* + BC^*), \quad s_2 = -\frac{1}{2}(AD^* - BC^*), \quad s_3 = \frac{i}{2}(AC^* - BD^*).$$

We find the expression for the invariant of Ψ^a:

$$S_0^2 - S_1^2 - S_2^2 - S_1^3 = \frac{1}{16}(AA^*CC^* + BB^*DD^* + ADB^*C^* + A^*D^*BC),$$

which can be rewritten as a product

$$S^a S_a = \frac{1}{16}(AC^* + BD^*)(A^*C + B^*D) = \frac{1}{16}|AC^* + BD^*|^2 \geq 0,$$

which means that the vector is time-like.

Bearing in mind that $\Psi^0 > 0$, we conclude that the 4-vector Ψ^a can be considered as a Stokes (Mueller) 4-vector of partially polarized light. The complex 3-vector s is not isotropic:

$$s^2 = -\frac{1}{4}(\xi^1\eta_{\dot{1}}^* + \xi^2\eta_{\dot{2}}^*)^2 = -\frac{1}{4}(AC^* + BD^*)^2 \neq 0.$$

With the use of the explicit form of the (imaginary) pseudo-vector $\tilde{\Psi}^a$,

$$\tilde{S}^0 = \frac{1}{4i}[(AA^* - DD^*) + (BB^* - CC^*)]\,, \tilde{S}^3 = \frac{1}{4i}[(AA^* - DD^*) - (BB^* - CC^*)]\,,$$

$$\tilde{S}^1 = \frac{1}{4i}[(AB^* + CD^*) + (BA^* + DC^*)]\,, \tilde{S}^2 = \frac{1}{4}[(AB^* + CD^*) - (BA^* + DC^*)]\,,$$

for its invariant we find

$$\tilde{\Psi}^a \tilde{\Psi}_a = \tilde{\Psi}_0^2 - \tilde{\Psi}_1^2 - \tilde{S}_2^2 - \tilde{S}_1^3 = \frac{1}{4}(AC^* + BD^*)(A^*C + B^*D) > 0\,.$$

Both these facts mean that the corresponding real 4-pseudo-vector $i\tilde{\Psi}^a$ cannot be considered as being of Stokes type.

This approach allows for the further development.

We can introduce the concept of the minimal Jones bispinor

$$\Psi^{min} = \begin{vmatrix} a\, e^{+i(\alpha-\beta)/2} \\ b\, e^{-i(\alpha-\beta)/2} \\ b\, e^{+i(s-t)/2} \\ a\, e^{-i(s-t)/2} \end{vmatrix} = \begin{vmatrix} a\, e^{+i\tau/2} \\ b\, e^{-i\tau/2} \\ b\, e^{+i\sigma/2} \\ a\, e^{-i\sigma/2} \end{vmatrix};$$

it provides us with the following Stokes 4-vector

$$S_0 = \frac{1}{2}(a^2 + b^2), \quad S_3 = \frac{1}{2}(a^2 - b^2),$$

$$S_1 = \frac{\cos\sigma - \cos\tau}{\sqrt{2(\cos^2\sigma + \cos^2\tau)}}, \quad S_2 = \frac{\cos\sigma + \cos\tau}{\sqrt{2(\cos^2\sigma + \cos^2\tau)}}.$$

The angular parameters τ_1, σ_1 are determined by the explicit formulas (there exist two different solutions)

$$\cos\sigma_1 = \frac{+S_1(S_1^2 + S_2^2) + S_2\sqrt{\left(S_1^2 + S_2^2\right)\left(S_0^2 - S_1^2 - S_2^2 - S_3^2\right)}}{(S_1^2 + S_2^2)\sqrt{S_0^2 - S_3^2}},$$

$$\sin\sigma_1 = \frac{S_2(S_1^2 + S_2^2) - S_1\sqrt{\left(S_1^2 + S_2^2\right)\left(S_0^2 - S_1^2 - S_2^2 - S_3^2\right)}}{(S_1^2 + S_2^2)\sqrt{S_0^2 - S_3^2}};$$

$$\cos\tau_1 = \frac{-S_1(S_1^2 + S_2^2) + S_2\sqrt{\left(S_1^2 + S_2^2\right)\left(S_0^2 - S_1^2 - S_2^2 - S_3^2\right)}}{(S_1^2 + S_2^2)\sqrt{S_0^2 - S_3^2}},$$

$$\sin\tau_1 = \frac{-S_2(S_1^2 + S_2^2) - S_1\sqrt{\left(S_1^2 + S_2^2\right)\left(S_0^2 - S_1^2 - S_2^2 - S_3^2\right)}}{(S_1^2 + S_2^2)\sqrt{S_0^2 - S_3^2}}.$$

Resolving this system, we obtain two different solutions

$$e^{i\sigma_2} = -e^{i\tau_1},\ e^{i\tau_2} = -e^{i\sigma_1};$$

$$e^{i\sigma_2/2} = ie^{i\tau_1/2}, e^{-i\sigma_2/2} = -ie^{-i\tau_1/2}; \quad e^{i\tau_2/2} = ie^{i\sigma_1/2}, e^{-i\tau_2/2} = -ie^{-i\sigma_1/2}.$$

So we get relationship between Ψ_2^{min} and Ψ_1^{min}:

$$\Psi_1^{min} = \begin{vmatrix} a\, e^{+i\tau_1/2} \\ b\, e^{-i\tau_1/2} \\ b\, e^{+i\sigma_1/2} \\ a\, e^{-i\sigma_1/2} \end{vmatrix}, \quad \Psi_2^{min} = \begin{vmatrix} a\, e^{+i\tau_2/2} \\ b\, e^{-i\tau_2/2} \\ b\, e^{+i\sigma_2/2} \\ a\, e^{-i\sigma_2/2} \end{vmatrix} = \begin{vmatrix} ia\, e^{+i\sigma_1/2} \\ -ib\, e^{-i\sigma_1/2} \\ ib\, e^{+i\tau_1/2} \\ -ia\, e^{-i\tau_1/2} \end{vmatrix},$$

they are related as follows

$$\Psi_2^{min} = -\begin{vmatrix} e^{-(\tau_1-\sigma_1)/2} & 0 & & 0 \\ 0 & -ie^{+(\tau_1-\sigma_1)/2} & 0 & \dfrac{b}{a} \\ 0 & 0 & ie^{+(\tau_1-\sigma_1)/2} & 0 \\ 0 & 0 & 0 & -ie^{-(\tau_1-\sigma_1)/2} \end{vmatrix} \Psi_1^{min}. \qquad (2)$$

Now let us turn to Stokes tensor, it turned out that this tensor depends on a more complicated bispinor than minimal one:

$$\Psi = \begin{vmatrix} e^{i\Gamma} & 0 & 0 & 0 \\ 0 & e^{i\Gamma} & 0 & \\ 0 & 0 & e^{-i\Gamma} & 0 \\ 0 & 0 & 0 & e^{-i\Gamma} \end{vmatrix} \Psi^{min} = \exp(i\Gamma\gamma^5)\,\Psi^{min}, \quad \Gamma = \frac{\alpha - t}{4} + \frac{\beta - s}{4}.$$

Correspondingly, the components of the Stokes tensor read

$$S^{01} = -\frac{a^2\sin(\alpha-t) + b^2\sin(\beta-s)}{2}, S^{23} = \frac{a^2\cos(\alpha-t) + b^2\cos(\beta-s)}{2},$$
$$S^{02} = -\frac{a^2\cos(\alpha-t) - b^2\cos(\beta-s)}{2}, S^{31} = -\frac{a^2\sin(\alpha-t) - b^2\sin(\beta-s)}{2}, \qquad (3)$$
$$S^{03} = \frac{-ab\sin(\alpha-s) + ab\sin(\beta-t)}{2}, S^{12} = \frac{ab\cos(\alpha-s) - ab\cos(\beta-t)}{2};$$

this tensor is parameterized by six parameters $\{a, b; \alpha - t, \beta - s\}$ and $\alpha - s, \beta - t$. The Stokes 4-vector is parameterized by four parameters $\{a, b, \tau = \alpha - \beta, \sigma = s - t\}$.

The four tensor components from (3) may be presented differently through S_a and angular variables $\rho = \alpha - t,\ \delta = \beta - s$:

$$S^{01} = -\frac{1}{2}[(S_0 + S_3)\sin\rho + (S_0 - S_3)\sin\delta],$$
$$S^{23} = \frac{1}{2}[(S_0 + S_3)\cos\rho + (S^0 - S^3)\cos\delta],$$
$$S^{02} = -\frac{1}{2}[(S_0 + S_3)\cos\rho - (S^0 - S^3)\cos\delta], \qquad (4)$$
$$S^{31} = -\frac{1}{2}[(S_0 + S_3)\sin\rho - (S^0 - S^3)\sin\delta];$$

the inverse formulas for (4) have the form (there arise two possibilities):

$$\rho_1 = arg\ \left(\frac{S^{23} - S^{02}}{S_0 + S_3} - i\frac{S^{31} + S^{01}}{S_0 + S_3}\right) \in [0, 2\pi], \quad \delta_1 = \rho_1 - (\tau + \sigma),$$

$$\delta_2 = arg\ (\frac{S^{23}+S^{02}}{S_0-S_3}+i\frac{S^{31}-S^{01}}{S_0-S_3})\in[0,2\pi], \quad \rho_2=\delta_2+(\tau+\sigma).$$

With the help of (2), one can express the angular variables $\tau,\ \sigma$ through Stokes vector S_a.

The two formulas below permit to calculate the components S^{03} and S^{12} through four components S^{01}, S^{31}, S^{02}, S^{23} and the known $S_0,\ S_1,\ S_2,\ S_3$

$$S^{03}=\frac{S_1(S^{01}-S^{31})+S_2(S^{02}+S^{23})}{S_0-S_3}, \quad S^{12}=\frac{S_2(S^{31}-S^{01})+S_1(S^{02}+S^{23})}{S_0-S_3}.$$

Numerical simulation

Example 1. For given $a,\ b,\ \tau,\ \sigma:\ a=1,\ b=2,\ \tau=\frac{\pi}{3},\ \sigma=\frac{\pi}{6}$ we find

$$S_0=\frac{5}{2}, \quad S_3=-\frac{3}{2}, \quad S_1=\frac{1}{2}(\sqrt{3}-1), \quad S_2=\frac{1}{2}(1-\sqrt{3});$$

thus, the Stokes 4-vector is

$$(S_0,S_1,S_2,S_3)\approx(2.5,+0.366,-0.366,-1.5).$$

In order to fix the Stokes tensor, we should add two parameters; let they be

$$\delta=\frac{5\pi}{12}, \ \rho=-\frac{\pi}{12};$$

so we find the first four components

$$S^{01}=-\frac{5+3\sqrt{3}}{4\sqrt{2}}, \ S^{02}=\frac{3\sqrt{3}-5}{4\sqrt{2}}, S^{23}=\frac{5\sqrt{3}-3}{4\sqrt{2}}, \ S^{31}=\frac{3+5\sqrt{3}}{4\sqrt{2}},$$

then find two remaining ones

$$S^{03}=-\frac{1}{2}\sqrt{\frac{3}{2}}(\sqrt{3}-1), S^{12}=-\frac{\sqrt{3}-1}{2\sqrt{2}}.$$

Thus, the Stokes tensor is given as

$$S^{01}=-1.80244, \quad S^{23}=1.0006,$$
$$S^{02}=0.0346752, \quad S^{31}=2.06126,$$
$$S^{03}=-0.448288, \quad S^{12}=-0.258819.$$

Example 2. Let $a=1,\ b=\frac{1}{2},\ \tau=\frac{\pi}{4},\ \sigma=\frac{\pi}{8};$ then the Stokes 4-vector is

$$S_0=\frac{5}{8}, \quad S_1=\frac{1}{8}(2\cos(\frac{\pi}{8})-\sqrt{2}), \quad S_2=\frac{1}{8}(2\sin(\frac{\pi}{8})-\sqrt{2}), \quad S_3=\frac{3}{8}.$$

Thus, the components of Stoke vector are

$$S_0=0.625,\ S_1\approx0.054,\ S_2\approx-0.081,\ S_3=0.375.$$

In order to fix the Stokes tensor, we should add two parameters, let they be

$$\delta=\frac{7\pi}{24}, \ \rho=0.$$

Further we obtain

$$S^{01}=-\frac{1}{8}\cos\frac{5\pi}{24}, \ S^{02}=\frac{1}{8}(\sin\frac{5\pi}{24}-4), \quad S^{23}=\frac{1}{8}(4+\sin\frac{5\pi}{24}), \ S^{31}=\frac{1}{8}\cos\frac{5\pi}{24},$$

and remaining ones

$$S^{03} = \frac{1}{8}(-1 - \sqrt{2}\sin\frac{5\pi}{24} + \sqrt{2}\cos\frac{5\pi}{24}),\ S^{12} = \frac{1}{8}(\sqrt{3} - \sqrt{2}\sin\frac{5\pi}{24} - \sqrt{2}\cos\frac{5\pi}{24}).$$

So the Stokes tensor is given as

$$S^{01} = -0.099, \quad S^{23} \approx 0.576,$$
$$S^{02} = -0.423, \quad S^{31} = 0.099,$$
$$S^{03} = -0.092, \quad S^{12} = -0.031.$$

These examples prove correctness of the performed study.

References

1. Red'kov, V.M. Majorana-Oppenheimer approach to Maxwell electrodynamics. Part I. Minkowski space / V.M. Red'kov, N.G. Tokarevskaya, G.J. Spix // Advances in Applied Clifford Algebras. − 2012. − Vol. 22, no 4. − P. 1129–1149.

2. Spinor Structures in Geometry and Physics / O.V. Veko, E.M. Ovsiyuk, A. Oana [et al.]. − New York : Nova Science Publishers Inc., 2015. – 415 p.

3. Spinor methods in the theory of group, and polarization optics / O.V. Veko, E.M. Ovsiyuk, V.V. Kisel, V.M. Red'kov. – Minsk : Belarus Science, 2019. – 409 p. (in Russian).

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SPIN 1/2 PARTICLE WITH ANOMALOUS MAGNETIC MOMENT AND POLARIZABILITYIN THE EXTERNAL MAGNETIC FIELD

In the present paper, we examine the Dirac-like equation for a spin 1/2 particle with two additional characteristics, anomalous magnetic moment μ and polarizability σ in presence of external uniform magnetic field. After separating the variables, we derive the system of four differential equations in the polar coordinate. To resolve the system of equations, we apply the method by Fedorov – Gronskiy. According to this approach, four polar components are expressed through only two different functions, the last reduce to the confluent hypergeometric equation; at this there arises a definite quantization rule due to the presence of the uniform magnetic field. We have constructed two types of the wave functions, the corresponding energy spectra are found in analytical form.

Keywords: Dirac-like equations, two additional, electromagnetic characteristics, external magnetic fields, projective operators, exact solutions, generalized energy spectra.

In the paper [1], within the general method by Gel'fand – Yaglom [2], starting with the extended set of representations of the Lorentz group, it was constructed a generalized equation for a spin 1/2 particle with two additional characteristics (concerning general formalism see in [3], [4]). After eliminating the accessory variables of the complete wave function, it was derived the generalized Dirac-like equation, the last includes two additional interaction terms which are