and remaining ones

$$S^{03}=\frac{1}{8}(-1-\sqrt{2}\sin\frac{5\pi}{24}+\sqrt{2}\cos\frac{5\pi}{24}),\ S^{12}=\frac{1}{8}(\sqrt{3}-\sqrt{2}\sin\frac{5\pi}{24}-\sqrt{2}\cos\frac{5\pi}{24}).$$

So the Stokes tensor is given as

$$S^{01}=-0.099,\quad S^{23}\approx 0.576,$$
$$S^{02}=-0.423,\quad S^{31}=0.099,$$
$$S^{03}=-0.092,\quad S^{12}=-0.031.$$

These examples prove correctness of the performed study.

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## SPIN 1/2 PARTICLE WITH ANOMALOUS MAGNETIC MOMENT AND POLARIZABILITYIN THE EXTERNAL MAGNETIC FIELD

In the present paper, we examine the Dirac-like equation for a spin 1/2 particle with two additional characteristics, anomalous magnetic moment μ and polarizability σ in presence of external uniform magnetic field. After separating the variables, we derive the system of four differential equations in the polar coordinate. To resolve the system of equations, we apply the method by Fedorov – Gronskiy. According to this approach, four polar components are expressed through only two different functions, the last reduce to the confluent hypergeometric equation; at this there arises a definite quantization rule due to the presence of the uniform magnetic field. We have constructed two types of the wave functions, the corresponding energy spectra are found in analytical form.

**Keywords:** Dirac-like equations, two additional, electromagnetic characteristics, external magnetic fields, projective operators, exact solutions, generalized energy spectra.

In the paper [1], within the general method by Gel'fand – Yaglom [2], starting with the extended set of representations of the Lorentz group, it was constructed a generalized equation for a spin 1/2 particle with two additional characteristics (concerning general formalism see in [3], [4]). After eliminating the accessory variables of the complete wave function, it was derived the generalized Dirac-like equation, the last includes two additional interaction terms which are interpreted as related to anomalous magnetic moment and a second additional characteristics:

$$\{\gamma^c i(\partial_c + ieA_c) + \frac{e\mu}{2M} j^{[ab]} F_{[ab]} + \frac{e\sigma}{2M^2} \gamma^c i(\partial_c + ieA_c) j^{[ab]} F_{[ab]} - M\}\Psi = 0; \qquad (1)$$

the parameter $\mu$ corresponds to anomalous magnetic moment of a spin 1/2 particle, and the second parameter $\sigma$ looks as related to a polarizability of the particle.

Let us consider this equation in presence of the uniform magnetic field. We will apply the cylindrical coordinates and the tetrad formalism. Let the field be oriented along the axis $z$, $A_\phi = +eBr^2/2,\ F_{12} = B$. Then the above equation (1) takes on the form

$$\{[\gamma^0 i\partial_t + \gamma^1(\partial_r + \frac{1}{2r}) + \frac{\gamma^2}{r} + (i\partial_\phi - eBr^2/2 + ij^{12}) +$$

$$+\gamma^3 i\partial_z](1 + \frac{e\sigma}{M^2} j^{12} F_{12}) + \frac{e\mu}{M} j^{12} F_{12} - M\}\Psi = 0. \qquad (2)$$

We will apply the following substitution for the wave function

$$\Psi = \frac{1}{\sqrt{r}} e^{-i\epsilon t} e^{im\phi} e^{ikz} \left|\begin{matrix} f_1(r) \\ f_2(r) \\ f_3(r) \\ f_4(r) \end{matrix}\right| = \frac{1}{\sqrt{r}} e^{-i\epsilon t} e^{im\phi} e^{ikz} F(r).$$

Let us simplify the notations

$$eB \Rightarrow B, \quad eF_{12} \Rightarrow +B, \quad e\mu \Rightarrow \mu, \quad e\sigma \Rightarrow \sigma\,,$$

$$a_{m+1/2} = \frac{d}{dr} + \frac{m + 1/2 + Br^2/2}{r}, \quad b_{m-1/2} = \frac{d}{dr} - \frac{m - 1/2 + Br^2/2}{r},$$

the equation (2) leads to

$$-a_{m+1/2} f_4(r)(\frac{B\sigma}{2M^2} + i) + f_3(r)(k+\epsilon)(1 - \frac{iB\sigma}{2M^2}) + f_1(r)(-\frac{iB\mu}{2M} - M) = 0,$$

$$b_{m-1/2} f_3(r)(\frac{B\sigma}{2M^2} - i) + f_4(r)(\epsilon - k)(1 + \frac{iB\sigma}{2M^2}) + f_2(r)(+\frac{iB\mu}{2M} - M) = 0,$$

$$a_{m+1/2} f_2(r)(\frac{B\sigma}{2M^2} + i) + f_1(r)(\epsilon - k)(1 - \frac{iB\sigma}{2M^2}) + f_3(r)(-\frac{iB\mu}{2M} - M) = 0,$$

$$-b_{m-1/2} f_1(r)(\frac{B\sigma}{2M^2} - i) + f_2(r)(\epsilon + k)(1 + \frac{iB\sigma}{2M^2}) + f_4(r)(+\frac{iB\mu}{2M} - M) = 0.$$

In order to resolve this system, we will apply the method by Fedorov – Gronskiy [5]. It is based on the use of projective operators related to the third spin projection

$$Y = ij^{12} = \left|\begin{matrix} 1/2 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & -1/2 \end{matrix}\right|; \quad P_+ = \left|\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}\right|, \quad P_- = \left|\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}\right|;$$

according to this approach, each projective constituent is determined through one function:

$$\Psi_{+}(r) = \begin{vmatrix} f_1 \\ 0 \\ f_3 \\ 0 \end{vmatrix} F_1(r), \quad \Psi_{-}(r) = \begin{vmatrix} 0 \\ f_2 \\ 0 \\ f_4 \end{vmatrix} F_2(r).$$

We impose differential constraints that permit us to transform all equations into algebraic ones:

$$a_{m+1/2} F_2(r) = C_1 F_1, \quad b_{m-1/2} F_1(r) = C_2 F_2,$$

taking into account these constraints we get the algebraic system

$$-C_1(\frac{B\sigma}{2M^2} + i) f_4 + (k+\epsilon)(1 - \frac{iB\sigma}{2M^2}) f_3 + (-\frac{iB\mu}{2M} - M) f_1 = 0,$$

$$C_2(\frac{B\sigma}{2M^2} - i) f_3 + (\epsilon - k)(1 + \frac{iB\sigma}{2M^2}) f_4 + (+\frac{iB\mu}{2M} - M) f_2 = 0,$$

$$C_1(\frac{B\sigma}{2M^2} + i) f_2 + (\epsilon - k)(1 - \frac{iB\sigma}{2M^2}) f_1 + (-\frac{iB\mu}{2M} - M) f_3 = 0,$$

$$-C_2(\frac{B\sigma}{2M^2} - i) f_1 + (\epsilon + k)(1 + \frac{iB\sigma}{2M^2}) f_2 + (+\frac{iB\mu}{2M} - M) f_4 = 0.$$

Without loss of generality, we can equate two parameters, $C_2 = C_1 = C$, so obtaining

$$(a_{m+1/2} b_{m-1/2} - C^2) F_1(r) = 0, \quad (b_{m-1/2} a_{m+1/2} - C^2) F_2(r) = 0; \qquad (3)$$

then the above algebraic system reads simpler

$$\begin{aligned} &-(\frac{iB\mu}{2M} + M) f_1 + 0 \cdot f_2 + (k+\epsilon)(1 - \frac{iB\sigma}{2M^2}) f_3 - C(\frac{B\sigma}{2M^2} + i) f_4 = 0, \\ &0 \cdot f_1 + (\frac{iB\mu}{2M} - M) f_2 + C(\frac{B\sigma}{2M^2} - i) f_3 + (\epsilon - k)(1 + \frac{iB\sigma}{2M^2}) f_4 = 0, \\ &(\epsilon - k)(1 - \frac{iB\sigma}{2M^2}) f_1 + C(\frac{B\sigma}{2M^2} + i) f_2 - (\frac{iB\mu}{2M} + M) f_3 + 0 \cdot f_4 = 0, \\ &-C(\frac{B\sigma}{2M^2} - i) f_1 + (\epsilon + k)(1 + \frac{iB\sigma}{2M^2}) f_2 + 0 \cdot f_3 + (\frac{iB\mu}{2M} - M) f_4 = 0. \end{aligned} \qquad (4)$$

In explicit form the equations (3) read

$$\frac{d^2 F_1}{dr^2} + \frac{1}{r}\frac{dF_1}{dr} + [-\frac{1}{4}B^2 r^2 - \frac{1}{2}B - mB - C^2 - \frac{(m-1/2)^2}{r^2}] F_1 = 0,$$

$$\frac{d^2 F_2}{dr^2} + \frac{1}{r}\frac{dF_2}{dr} + [-\frac{1}{4}B^2 r^2 - \frac{1}{2}B - mB - C^2 - \frac{(m+1/2)^2}{r^2}] F_2 = 0.$$

Let us transform them to the variable, $x = -Br^2/2$. These equations are related by simple symmetr $B \Rightarrow -B,\ m \Rightarrow -m,\ F_1 \Rightarrow F_2$; so it suffices to solve the equation for $F_1(x) = x^A e^{Dx} f_1(x)$:

$$f'' + (\frac{2A+1}{x} + 2D) f' + \frac{A^2}{x^2} f - \frac{1}{4}\frac{(1/2-m)^2}{x^2} f +$$

$$+ D^2 f - \frac{1}{4} f + \frac{(2A+1)D}{x} f + \frac{1}{4}\frac{1 + 2m + 2C^2/B}{x} f = 0.$$

In order to have finite solutions, we should use $A = +\frac{|m-1/2|}{2}, D = +1/2 \quad (let\ B > 0)$. In this way, in the variable $y = -x$ we get a confluent hypergeometric equation with parameters

$$c = |m - 1/2| + 1, \quad a = \frac{|m-1/2| + m + 1/2}{2} + \frac{C^2}{2B} + \frac{1}{2}.$$

The polynomial condition $a = -n$ gives the following quantization rule

$$C^2 = -2B\left(n + \frac{|m-1/2| + m + 1/2}{2} + \frac{1}{2}\right), \quad n = 0, 1, 2, ...$$

Let us turn to the algebraic system (4). It is convenient to apply dimensionless quantities

$$\frac{\epsilon}{M} = E, \quad \frac{k}{M} = K, \quad \frac{C}{M} = c, \quad b = \frac{B^2}{2M}, \quad \frac{B\sigma}{2M^2} = ib\sigma, \quad \frac{iB\mu}{2M^2} = ib\mu;$$

then the system (4) in matrix form reads

$$\begin{vmatrix} -(ib\mu+1) & 0 & (E+K)(1-ib\sigma) & -c(b\sigma+i) \\ 0 & (ib\mu-1) & c(b\sigma-i) & (E-K)(1+ib\sigma) \\ (E-K)(1-ib\sigma) & c(b\sigma+i) & -(ib\mu+1) & 0 \\ -c(b\sigma-i) & (E+K)(1+ib\sigma) & 0 & (ib\mu-1) \end{vmatrix} \begin{vmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{vmatrix} = 0. \qquad (5)$$

From vanishing its determinant, we derive a bi-quadratic equation

$$\det A = b^4 \left[ \left( E^2 - K^2 + c^2 \right) \sigma^2 - \mu^2 \right]^2 + \left( E^2 - K^2 + c^2 - 1 \right)^2 +$$

$$+ b^2 \left\{ 2\left( E^2 - K^2 - c^2 + 1 \right) \mu^2 + 2\left[ \left( E^2 - K^2 + c^2 \right)^2 + E^2 - K^2 - c^2 \right] \sigma^2 + 8\left( E^2 - K^2 \right) \sigma\mu \right\} = 0.$$

For parameters $E_{1,2} > 0$, we obtain expressions

$$E_{1,2} = E_{\pm} = \frac{1}{1 + b^2\sigma^2} [\pm 2b(\sigma + \mu)\sqrt{c^2\left(1 + b^2\sigma^2\right)^2 - \left(b^2\mu\sigma - 1\right)^2} +$$

$$+ \left( \left( -c^2 + K^2 \right)\sigma^4 + \sigma^2\mu^2 \right) b^4 + \left( \left( -1 - 2c^2 + 2K^2 \right)\sigma^2 - 4\sigma\mu - \mu^2 \right) b^2 - c^2 + 1 + K^2 ]^{1/2}.$$

Substituting expression for $E_{1,2}$ in the system (5) we can find two types of the wave functions. The energy spectra depend in a complicated way on additional characteristics; by this reason these spectra may be studied numerically. By physical reason, two additional parameters should are imaginary; only then we get the physically interpretable positive energies.

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## GEOMETRICAL MODELLING ON THE MEDIA IN ELECTRODYNAMICS

This paper includes the following items: Riemannian geometry and Maxwell theory; Maxwell equations in Riemannian space and effective media; metrical tensor $g_{\alpha\beta}(x)$ and constitutive relations; inverse constitutive equations; geometric simulation of inhomogeneous media; geometrical modeling of anisotropic uniform media; the moving medium and anisotropy.

**Keywords:** Maxwell equations, Riemannian space, effective media, geometrical modeling.

**Introduction.** Note that Gordon [1] was the first interested in trying to describe dielectric media by an effective metrics. Gordon tried to use a gravitational field to simulate a dielectric medium. The idea was taken up and developed by Tamm and Mandel'stam [2]−[4], and by many others.

Let us start with the Maxwell equations in Minkowski space for the uniform medium

$$\mathrm{div}\mathbf{B}=0,\quad \mathrm{rot}\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t},\quad \epsilon\epsilon_0\,\mathrm{div}\mathbf{E}=\rho,\quad \frac{1}{\mu\mu_0}\mathrm{rot}\mathbf{B}=\mathbf{J}+\epsilon\epsilon_0\frac{\partial\mathbf{E}}{\partial t}. \tag{1}$$

With the use of constitutive relations $\mathbf{H}=\dfrac{\mathbf{B}}{\mu\mu_0},\quad \mathbf{D}=\epsilon\epsilon_0\mathbf{E}$, eqs. (1) can be written as

$$\mathrm{div}c\mathbf{B}=0,\quad \mathrm{rot}\mathbf{E}=-\frac{\partial c\mathbf{B}}{\partial x^0},\quad \mathrm{div}\mathbf{D}=j^0,\quad \mathrm{rot}\frac{\mathbf{H}}{c}=\frac{\mathbf{J}}{c}+\frac{\partial\mathbf{D}}{\partial x^0}\ \ (x^0=ct). \tag{2}$$

We represent the electric displacement $\mathbf{D}$ and the magnetic field $\mathbf{H}$ by the antisymmetric tensor $H^{ik}$, the electric $\mathbf{E}$ and the magnetic induction $\mathbf{B}$ are accounted for by the tensor $F^{ik}$:

$$(F^{\alpha\beta})=\begin{vmatrix}0 & -E^1 & -E^2 & -E^3\\ E^1 & 0 & -cB^3 & cB^2\\ E^2 & cB^3 & 0 & -cB^1\\ E^3 & -cB^2 & cB^1 & 0\end{vmatrix},\quad (H^{\alpha\beta})=\begin{vmatrix}0 & -D^1 & -D^2 & -D^3\\ D^1 & 0 & -H^3/c & H^2/c\\ D^2 & H^3/c & 0 & -H^1/c\\ D^3 & -H^2/c & H^1/c & 0\end{vmatrix};$$

where $E^i=-E_i,\ D^i=-D_i,\ B^i=+B_i,\ H^i=+H_i,\ j^a=(\rho,\mathbf{J}/c)$. Then eqs. (2) may be presented in relativistic covariant tensor form

$$\partial_a F_{bc}+\partial_b F_{ca}+\partial_c F_{ab}=0,\quad \partial_b H^{ba}=j^a. \tag{3}$$