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UDC 537.8

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## GEOMETRICAL MODELLING ON THE MEDIA IN ELECTRODYNAMICS

This paper includes the following items: Riemannian geometry and Maxwell theory; Maxwell equations in Riemannian space and effective media; metrical tensor $g_{\alpha\beta}(x)$ and constitutive relations; inverse constitutive equations; geometric simulation of inhomogeneous media; geometrical modeling of anisotropic uniform media; the moving medium and anisotropy.

**Keywords:** Maxwell equations, Riemannian space, effective media, geometrical modeling.

**Introduction.** Note that Gordon [1] was the first interested in trying to describe dielectric media by an effective metrics. Gordon tried to use a gravitational field to simulate a dielectric medium. The idea was taken up and developed by Tamm and Mandel'stam [2]−[4], and by many others.

Let us start with the Maxwell equations in Minkowski space for the uniform medium

$$\mathrm{div}\mathbf{B} = 0, \quad \mathrm{rot}\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \epsilon\epsilon_0 \,\mathrm{div}\mathbf{E} = \rho, \quad \frac{1}{\mu\mu_0}\mathrm{rot}\mathbf{B} = \mathbf{J} + \epsilon\epsilon_0 \frac{\partial \mathbf{E}}{\partial t}. \tag{1}$$

With the use of constitutive relations $\mathbf{H} = \dfrac{\mathbf{B}}{\mu\mu_0}, \quad \mathbf{D} = \epsilon\epsilon_0 \mathbf{E}$, eqs. (1) can be written as

$$\mathrm{div}c\mathbf{B} = 0, \quad \mathrm{rot}\,\mathbf{E} = -\frac{\partial c\mathbf{B}}{\partial x^0}, \quad \mathrm{div}\mathbf{D} = j^0, \quad \mathrm{rot}\frac{\mathbf{H}}{c} = \frac{\mathbf{J}}{c} + \frac{\partial \mathbf{D}}{\partial x^0} \;\; (x^0 = ct). \tag{2}$$

We represent the electric displacement $\mathbf{D}$ and the magnetic field $\mathbf{H}$ by the antisymmetric tensor $H^{ik}$, the electric $\mathbf{E}$ and the magnetic induction $\mathbf{B}$ are accounted for by the tensor $F^{ik}$:

$$(F^{\alpha\beta}) = \begin{vmatrix} 0 & -E^1 & -E^2 & -E^3 \\ E^1 & 0 & -cB^3 & cB^2 \\ E^2 & cB^3 & 0 & -cB^1 \\ E^3 & -cB^2 & cB^1 & 0 \end{vmatrix}, \quad (H^{\alpha\beta}) = \begin{vmatrix} 0 & -D^1 & -D^2 & -D^3 \\ D^1 & 0 & -H^3/c & H^2/c \\ D^2 & H^3/c & 0 & -H^1/c \\ D^3 & -H^2/c & H^1/c & 0 \end{vmatrix};$$

where $E^i = -E_i, D^i = -D_i, B^i = +B_i, H^i = +H_i, j^a = (\rho, \mathbf{J}/c)$. Then eqs. (2) may be presented in relativistic covariant tensor form

$$\partial_a F_{bc} + \partial_b F_{ca} + \partial_c F_{ab} = 0, \quad \partial_b H^{ba} = j^a. \tag{3}$$

For the vacuum case, the constitutive relations $\mathbf{D}=\epsilon_0\mathbf{E},\ \ \mathbf{H}=\dfrac{1}{\mu_0}\mathbf{B},$ read in tensor form as follows $H^{ab}(x)=\epsilon_0 F^{ab}(x),$ and eqs. (3) contain only one tensor

$$\partial_a F_{bc}+\partial_b F_{ca}+\partial_c F_{ab}=0, \quad \partial_b F^{ba}=\frac{1}{\epsilon_0} j^a .$$

The situation is quite different in presence of media. Even for simplest case of the uniform medium, relativizing the above constitutive equations $\mathbf{D}=\epsilon_0\epsilon\mathbf{E},\ \ \mathbf{H}=\dfrac{1}{\mu_0\mu}\mathbf{B}$ requires a subsidiary $(4\times 4)$ -matrix with presumed properties of a 2-rank tensor:

$$\eta^{am}=\sqrt{\epsilon}\begin{vmatrix} 1/k & 0 & 0 & 0 \\ 0 & -k & 0 & 0 \\ 0 & 0 & -k & 0 \\ 0 & 0 & 0 & -k \end{vmatrix}, \quad k=\frac{1}{\sqrt{\epsilon\mu}}, \quad H^{ab}=\epsilon_0\eta^{am}\eta^{bn}F_{mn}.$$

A class of linear inhomogeneous electromagnetic media characterized by a 4-rank tensor may be postulated as [3], [4]:

$$H^{ab}(x)=\epsilon_0\Delta^{abmn}(x)F_{mn}(x), \quad \Delta^{abmn}(x)=-\Delta^{bamn}(x)=-\Delta^{abnm}(x),$$

When extending Maxwell theory to a space-time with non-Euclidean geometry, which describes gravity according to General Relativity, one must change previous equations to a more general form. In particular, the vacuum Maxwell equations read

$$\nabla_\alpha f_{\beta\gamma}+\nabla_\beta f_{\gamma\alpha}+\nabla_\gamma f_{\alpha\beta}=0, \quad \nabla_\beta h^{\beta\alpha}=j^\alpha, \quad h_{\alpha\beta}=\epsilon_0 f_{\alpha\beta},$$

where $\nabla_\beta$ stands for the covariant derivative. In order to distinguish formulas referring to a flat and curved models we will use small letters to designates electromagnetic tensors in curved model, $f_{ab}$ and $h^{ab}$.

**1. Maxwell equations in Riemannian space and effective media.** Let us discuss the possibility to consider the vacuum Maxwell equations in a curved space-time as Maxwell equations in flat space-time but specified for an effective medium, the properties of which are determined by metrical structure of the initial curved model $g_{\alpha\beta}(x)$. We will restrict ourselves to the case of curved space-time models which are parameterized by some quasi-Cartesian coordinates.

Vacuum Maxwell equations in a Riemannian space-time, parameterized by some quasi-Cartesian coordinates can be brought to the form [5], [6]

$$\partial_a f_{bc}+\partial_b f_{ca}+\partial_c f_{ab}=0, \quad \frac{1}{\sqrt{-g}}\partial_b\sqrt{-g}\, f^{ba}=\frac{1}{\epsilon_0} j^a . \qquad (4)$$

Indeed, one can immediately see that introducing new variables

$$\sqrt{-g}\, j^a\to j^a, \quad f_{ab}\to F_{ab}, \quad \epsilon_0\sqrt{-g}\, g^{am}(x)g^{bn}(x)f_{mn}(x)\to H^{ba},$$

equations (4) in the curved space can be re-written as Maxwell equations for the flat space but in a medium:

$$\partial_a F_{bc}+\partial_b F_{ca}+\partial_c F_{ab}=0, \quad \partial_b H^{ba}=\frac{1}{\epsilon_0} j^a .$$

Relations playing the role of constitutive equations are determined by the metrical structure of the geometrical model:

$$H^{\beta\alpha}(x) = \epsilon_0 \left[ \sqrt{-g(x)}\, g^{\alpha\rho}(x) g^{\beta\sigma}(x) \right] F_{\rho\sigma}(x). \tag{5}$$

**2. Metrical tensor and constitutive relations.** Let us consider the constitutive equations for electromagnetic fields which are generated by the metrical structure of the space-time model. For an arbitrary metrical tensor $g_{\alpha\beta}(x)$ we may obtain a 3-dimensional form:

$$D^i = \epsilon_0 \epsilon^{ik}(x) E_k + \epsilon_0 c \alpha^{ik}(x) B_k, \quad H^i = \epsilon_0 c \beta^{ik}(x) E_k + \mu_0^{-1} (\mu^{-1})^{ik}(x) B_k.$$

Four dimensionless $(3\times 3)$-matrices $\epsilon^{ik}(x)$, $\alpha^{ik}(x)$, $\beta^{ik}(x)$, $(\mu^{-1})^{ik}(x)$ are not independent because they are bilinear functions of only 10 components of the symmetrical tensor $g_{\alpha\beta}(x)$.

After simple calculation, for these tensors one produces expressions

$$\epsilon^{ik}(x) = \sqrt{-g}\left( g^{00}(x) g^{ik}(x) - g^{0i}(x) g^{0k}(x) \right), \qquad (\mu^{-1})^{ik}(x) = \frac{1}{2}\sqrt{-g}\, \epsilon_{imn}\, g^{ml}(x) g^{nj}(x) \epsilon_{ljk},$$

$$\alpha^{ik}(x) = +\sqrt{-g}\, g^{ij}(x) g^{0l}(x) \epsilon_{ljk}, \quad \beta^{ik}(x) = -\sqrt{-g}\, g^{0j}(x) \epsilon_{jil}\, g^{lk}(x).$$

The tensor $\epsilon^{ik}(x)$ is evidently symmetrical; it is easy to demonstrate the same property for $(\mu^{-1})^{ik}(x)$. Indeed, we have

$$(\mu^{-1})^{ki}(x) = \frac{1}{2}\epsilon_{kmn}\, g^{ml}(x) g^{nj}(x) \epsilon_{lji},$$

making changes in mute indices, $m \leftrightarrow j$, $n \leftrightarrow l$, we get

$$(\mu^{-1})^{ki}(x) = \frac{1}{2}\epsilon_{kjl}\, g^{jn}(x) g^{lm}(x) \epsilon_{nmi} \epsilon_{imn}\, g^{lm}(x) g^{jn}(x) \epsilon_{ljk} = (\mu^{-1})^{ik}(x).$$

In the same manner, one can verify the identity $\beta^{ki}(x) = +\alpha^{ik}$:

$$\beta^{ki} = -g^{0j}(x) \epsilon_{jkl}\, g^{li}(x) = g^{il}(x) g^{0j}(x) \epsilon_{jlk} = +\alpha^{ik}.$$

So, the tensors obey the constraints:

$$\epsilon^{ik}(x) = +\epsilon^{ki}(x), \quad (\mu^{-1})^{ik}(x) = +(\mu^{-1})^{ki}(x), \quad \beta^{ki}(x) = \alpha^{ik};$$

they mean that the $(6\times 6)$-matrix defining constitutive equations is symmetrical

$$\begin{vmatrix} D^i(x) \\ H^i(x) \end{vmatrix} = \begin{vmatrix} \epsilon_0 \epsilon^{ik}(x) & \epsilon_0 c \alpha^{ik}(x) \\ \epsilon_0 c \beta^{ik}(x) & \mu_0^{-1} (\mu^{-1})^{ik}(x) \end{vmatrix} \begin{vmatrix} E_k(x) \\ B_k(x) \end{vmatrix}.$$

Making $(3+1)$-splitting in the metrical tensor

$$g^{\alpha\beta}(x) = \begin{vmatrix} g^{00} & (g^{0i}) = \bar{g} \\ (g^{i0}) = \bar{g} & (g^{ik}) = g \end{vmatrix}, \quad (\bar{g}^{\times})_{jk} \equiv g^{0l}(x) \epsilon_{ljk} = g^{l}(x) \epsilon_{ljk},$$

tensors $(\epsilon^{ik})$, $(\alpha^{ik})$, $(\beta^{ik})$ can be written in the form

$$\epsilon(x) = \sqrt{-g}\,[g^{00}(x) g(x) - \bar{g}(x)\cdot \bar{g}(x)], \;\; \alpha(x) = \sqrt{-g}\, g(x) \bar{g}^{\times}(x), \;\; \beta(x) = -\sqrt{-g}\, \bar{g}^{\times}(x) g(x).$$

Metrical tensors which are the most interesting in the context of General relativity [5] have a quasi-diagonal structure, then the effective constitutive relations simplify:

$$g^{\alpha\beta}(x)=\begin{vmatrix} g^{00} & 0 & 0 & 0 \\ 0 & g^{11} & g^{12} & g^{13} \\ 0 & g^{21} & g^{22} & g^{23} \\ 0 & g^{31} & g^{32} & g^{33} \end{vmatrix}, \quad \alpha(x)=0, \quad \beta(x)=0,$$

$$\epsilon(x)=\sqrt{-g}\, g^{00}(x)g(x), \quad (\mu^{ik})(x)=-\frac{1}{2}\sqrt{-g}\ \mathrm{Sp}[\tau_i\, g(x)\tau_k\, g(x)]. \qquad (6)$$

Explicit expressions for tensors $\epsilon^{ik}(x)$ and $(\mu^{-1})^{ik}(x)$ given by (6) are

$$(\epsilon^{ik})=\sqrt{-g}\, g^{00}\begin{vmatrix} g^{11} & g^{12} & g^{13} \\ g^{21} & g^{22} & g^{23} \\ g^{31} & g^{32} & g^{33} \end{vmatrix}, \quad ((\mu^{-1})^{ik})=\sqrt{-g}\begin{vmatrix} G^{11} & G^{12} & G^{13} \\ G^{21} & G^{22} & G^{23} \\ G^{31} & G^{32} & G^{33} \end{vmatrix},$$

where $G^{ik}(x)$ stand for (algebraic) co-factors to the elements $g^{ik}(x)$:

$$G^{ik}(x)=\begin{vmatrix} (g^{22}g^{33}-g^{23}g^{32}) & (g^{31}g^{23}-g^{21}g^{33}) & (g^{21}g^{32}-g^{22}g^{31}) \\ (g^{32}g^{13}-g^{33}g^{12}) & (g^{33}g^{11}-g^{31}g^{13}) & (g^{31}g^{12}-g^{32}g^{11}) \\ (g^{12}g^{23}-g^{13}g^{22}) & (g^{13}g^{21}-g^{11}g^{23}) & (g^{11}g^{22}-g^{12}g^{21}) \end{vmatrix}.$$

Therefore, two matrices, $\epsilon(x)$ and $\mu^{-1}(x)$, are not independent and obey the following constraint:

$$\epsilon(x)\mu^{-1}(x)=\frac{-g^{00}g_{00}}{\det g_{ik}}\begin{vmatrix} g^{11} & g^{12} & g^{13} \\ g^{21} & g^{22} & g^{23} \\ g^{31} & g^{32} & g^{33} \end{vmatrix}$$

$$\begin{vmatrix} (g^{22}g^{33}-g^{23}g^{32}) & (g^{31}g^{23}-g^{21}g^{33}) & (g^{21}g^{32}-g^{22}g^{31}) \\ (g^{32}g^{13}-g^{33}g^{12}) & (g^{33}g^{11}-g^{31}g^{13}) & (g^{31}g^{12}-g^{32}g^{11}) \\ (g^{12}g^{23}-g^{13}g^{22}) & (g^{13}g^{21}-g^{11}g^{23}) & (g^{11}g^{22}-g^{12}g^{21}) \end{vmatrix}=-I.$$

Thus, the metric tensors with quasi-diagonal structure effectively describe media with following constitutive relations (the sign minus may be eliminated by changing the notation)

$$\mathbf{D}=-\epsilon_0\epsilon(x)\mathbf{E}, \quad \mathbf{B}=\mu_0\mu(x)\mathbf{H}, \quad \mu(x)=-\epsilon(x),$$

$$(\epsilon^{ik})(x)=\sqrt{-g(x)}\, g^{00}(x)\begin{vmatrix} g^{11}(x) & g^{12}(x) & g^{13}(x) \\ g^{21}(x) & g^{22}(x) & g^{23}(x) \\ g^{31}(x) & g^{32}(x) & g^{33}(x) \end{vmatrix}.$$

**3. Geometrical modeling of the inhomogeneous media.** Let us consider a special form of the diagonal anisotropic metric

$$g_{\alpha\beta}=\begin{vmatrix} a^2 & 0 & 0 & 0 \\ 0 & -b_1^2 & 0 & 0 \\ 0 & 0 & -b_2^2 & 0 \\ 0 & 0 & 0 & -b_3^2 \end{vmatrix},$$

where $a^2,\ b_i^2$ are arbitrary numerical parameters. The constitutive equations generated by this geometry have the form $D^i=\epsilon_0\epsilon^{ik}E_k, \quad H^i=\mu_0^{-1}\mu^{ik}B_k$, where

$$(\epsilon^{ik}) = a^{-2}\begin{vmatrix} -b_1^{-2} & 0 & 0 \\ 0 & -b_2^{-2} & 0 \\ 0 & 0 & -b_3^{-2} \end{vmatrix}, \quad (\mu^{ik}) = \begin{vmatrix} b_2^{-2}b_3^{-2} & 0 & 0 \\ 0 & b_3^{-2}b_1^{-2} & 0 \\ 0 & 0 & b_1^{-2}b_2^{-2} \end{vmatrix},$$

or differently

$$D^1 = -\frac{\epsilon_0 E_1}{a^2 b_1^2}, \quad D^2 = -\frac{\epsilon_0 E_2}{a^2 b_2^2}, \quad D^3 = -\frac{\epsilon_0 E_3}{a^2 b_3^2}, \quad H^1 = \frac{B_1}{\mu_0 b_2^2 b_3^2}, \quad H^2 = \frac{B_2}{\mu_0 b_3^2 b_1^2}, \quad H^3 = \frac{B_3}{\mu_0 b_1^2 b_2^2}.$$

These equations should be compared with the physical ones

$$D^1 = -\epsilon_0\epsilon_1 E_1, \quad D^2 = -\epsilon_0\epsilon_2 E_2, \quad D^3 = -\epsilon_0\epsilon_3 E_3, \quad H^1 = \frac{B_1}{\mu_0\mu_1}, \quad H^2 = \frac{B_2}{\mu_0\mu_2}, \quad H^3 = \frac{B_3}{\mu_0\mu_3},$$

whence we obtain

$$\epsilon_1 = \frac{1}{a^2 b_1^2}, \quad \epsilon_2 = \frac{1}{a^2 b_2^2}, \quad \epsilon_3 = \frac{1}{a^2 b_3^2}, \quad \mu_1 = b_2^2 b_3^2, \quad \mu_2 = b_3^2 b_1^2, \quad \mu_3 = b_1^2 b_2^2.$$

One can readily obtain

$$\frac{\mu_1}{\epsilon_1} = \frac{\mu_2}{\epsilon_2} = \frac{\mu_3}{\epsilon_3} = (a^2 b_1^2 b_2^2 b_3^2) = -g, \quad -g = \sqrt{\frac{\mu_1^2 + \mu_2^2 + \mu_3^2}{\epsilon_1^2 + \epsilon_2^2 + \epsilon_2^2}}, \quad \frac{\mu_i}{\sqrt{\mu_1^2 + \mu_2^2 + \mu_3^2}} = \frac{\epsilon_i}{\sqrt{\epsilon_1^2 + \epsilon_2^2 + \epsilon_2^2}}.$$

The latter means that one may use four independent parameters $\epsilon, \mu, n_i$:

$$\epsilon_i = \epsilon n_i, \quad \mu_i = \mu n_i, \quad \mathbf{n}^2 = 1.$$

One can readily express $b_i^2$ in terms of $\mu_i$:

$$b_1^2 = \sqrt{\frac{\mu_2\mu_3}{\mu_1}} = \sqrt{\mu}\sqrt{\frac{n_2 n_3}{n_1}}, \quad b_2^2 = \sqrt{\frac{\mu_3\mu_1}{\mu_2}} = \sqrt{\mu}\sqrt{\frac{n_3 n_1}{n_2}}, \quad b_3^2 = \sqrt{\frac{\mu_1\mu_2}{\mu_3}} = \sqrt{\mu}\sqrt{\frac{n_1 n_2}{n_3}}. \tag{7}$$

In turn, from $a^2 b_1^2 b_2^2 b_3^2 = \mu/\epsilon$ it follows

$$a^2 = \frac{\mu}{\epsilon}\frac{1}{b_1^2 b_2^2 b_3^2} = \frac{1}{\epsilon\sqrt{\mu}}\frac{1}{\sqrt{n_1 n_2 n_3}}. \tag{8}$$

The formulas (7), (8) provide us with anisotropic metrical tensor

$$g_{ab}(x) = \frac{1}{\sqrt{\epsilon}}\begin{vmatrix} \frac{1}{\sqrt{\epsilon\mu}}\frac{1}{\sqrt{n_1 n_2 n_3}} & 0 & 0 & 0 \\ 0 & -\sqrt{\epsilon\mu}\sqrt{\frac{n_2 n_3}{n_1}} & 0 & 0 \\ 0 & 0 & -\sqrt{\epsilon\mu}\sqrt{\frac{n_3 n_1}{n_2}} & 0 \\ 0 & 0 & 0 & -\sqrt{\epsilon\mu}\sqrt{\frac{n_1 n_2}{n_3}} \end{vmatrix}.$$

**4. The moving medium and anisotropy.** Starting point is that in Minkowski approach to electrodynamics, the constitutive relations explicitly depend on the 4-velocity of the reference frame motion under a medium. Gordon [2], Tamm and Mandel'stam [3], [4] noticed that for a moving observer the constitutive relations can be expressed with the help of effective metric as follows:

$$H^{ab}(x)=\epsilon_0\Delta^{abmn}F_{mn},\ \Delta^{abmn}=\epsilon_0\frac{1}{\sqrt{\mu}}[g^{am}+(\epsilon\mu-1)u^a u^m]\frac{1}{\sqrt{\mu}}[g^{bn}+(\epsilon\mu-1)u^b u^n],$$

where $g^{ab}=\mathrm{diag}(+1,-1,-1,-1)$. Corresponding constitutive 3-dimensional tensors are (let us use the notation $\epsilon\mu-1=\gamma$)

$$\epsilon^{ik}=\frac{1}{\mu}\begin{vmatrix} (-1+\gamma u^1u^1-\gamma u^0u^0) & \gamma u^1u^2 & \gamma u^1u^3 \\ \gamma u^1u^2 & (-1+\gamma u^2u^2-\gamma u^0u^0) & \gamma u^2u^3 \\ \gamma u^3u^1 & \gamma u^3u^2 & (-1+\gamma u^3u^3-\gamma u^0u^0) \end{vmatrix},$$

$$(\mu^{-1})^{ik}=\frac{1}{\mu}\begin{vmatrix} (1-\gamma u^2u^2-\gamma u^3u^3) & \gamma u^1u^2 & \gamma u^1u^3 \\ \gamma u^1u^2 & (1-\gamma u^3u^3-\gamma u^1u^1) & \gamma u^2u^3 \\ \gamma u^3u^1 & \gamma u^3u^2 & (1-\gamma u^1u^1-\gamma u^2u^2) \end{vmatrix},$$

$$\alpha^{ik}=\frac{1}{\mu}\begin{vmatrix} 0 & -\gamma u^0u^3 & +\gamma u^0u^2 \\ +\gamma u^0u^3 & 0 & -\gamma u^0u^1 \\ -\gamma u^0u^2 & +\gamma u^0u^1 & 0 \end{vmatrix},\quad \beta^{ik}=\frac{1}{\mu}\begin{vmatrix} 0 & +\gamma u^0u^3 & -\gamma u^0u^2 \\ -\gamma u^0u^3 & 0 & +\gamma u^0u^1 \\ +\gamma u^0u^2 & -\gamma u^0u^1 & 0 \end{vmatrix}.$$

Further we deduce vector form for constitutive relations, applying the notation

$$V=v/c,\quad u^0=\frac{1}{\sqrt{1-V^2}},\quad u^i=\frac{V^i}{\sqrt{1-V^2}},$$

$$\mathbf{D}=\frac{\epsilon_0}{\mu}\mathbf{E}+\frac{\epsilon_0\gamma}{\mu}\frac{\mathbf{E}-(\mathbf{VE})\mathbf{V}}{1-V^2}+\frac{\epsilon_0 c\gamma}{\mu}\frac{\mathbf{V}\times\mathbf{B}}{1-V^2}, \tag{9}$$

$$\mathbf{H}=\frac{1}{\mu_0\mu}\mathbf{B}+\frac{\gamma}{\mu_0\mu}\frac{\mathbf{V}\times(\mathbf{V}\times\mathbf{B})}{1-V^2}+\frac{\epsilon_0 c\gamma}{\mu}\frac{\mathbf{V}\times\mathbf{E}}{1-V^2}. \tag{10}$$

Relations (9), (10) provide us with vector form of constitutive relations for the uniform medium moving with velocity $\mathbf{V}$. We may conclude that the motion is effectively equivalent to an anisotropic medium. Besides, these relations mean that the Maxwell relations in media depend explicitly on the velocity of the reference frame, therefore they are not invariant under the Lorentz transformation.

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