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S=0 PARTICLE WITH THE COX STRUCTURE AND POLARIZABILITY, IN PRESENCE OF MAGNETIC FIELD

In the present paper, we will specify the equations for a spin 0 particle with two additional characteristics, Cox structure and polarizability, to the cylindrical coordinates, and in presence of the external uniform magnetic field. After separating the variables, we derive the system of 5 first order differential equations in polar coordinate. This system may be reduced to a second order equation for a primary function, which is solved in terms of the confluent hypergeometric functions. We derive a formula for energies, it depends on two additional characteristics.

Keywords: spin 0 particle, Cox structure, polarizability, external magnetic field, exact solutions, generalized energy spectra.

Within the method by Gel'fand – Yaglom [1], [2], in [3] it was constructed a relativistic system of the first order equations for a spin 0 particle with two additional characteristics, Cox structure and polarizability. After eliminating the accessory variables of the complete wave function, it was derived the system of 5 equations, they include two additional interaction terms which are interpreted as related to Cox structure and polarizability [2]. In the present paper, we will specify this equations to the cylindrical coordinates, and in presence of the external uniform magnetic field. We start with the form of equations in any curvilinear coordinates; the tensor system of equations takes form

$$D_\alpha \Phi^\alpha - iM\Phi = 0, \quad D_\beta \Phi + ie\gamma F_{\alpha\beta}\Phi^\alpha + ie\sigma D^\alpha F_{\alpha\beta}\Phi - iM\Phi_\beta = 0;$$

note the notations $D_\alpha = \nabla_\alpha + ieA_\alpha$, $\hat{\partial}_\alpha = \partial_\alpha + ieA_\alpha$. In tetrad form, they read

$$\hat{\partial}_{(a)}\Phi^a + e^{(a)\alpha}{}_{;\alpha}\Phi_a - iM\Phi = 0, \qquad \partial_{(c)}\Phi + ie\gamma F_{ac}\Phi^a - iM\Phi_c +$$
$$+ie\sigma[F^a{}_c\hat{\partial}_{(a)}\Phi + (\partial_{(a)}F^a{}_c)\Phi + F^a{}_c e^\alpha_{(a);\alpha}\Phi + F^{ab}(e_{(b)\beta;\alpha}e^\beta_{(c)}e^\alpha_{(a)})\Phi] = 0,$$

where we apply the notation $e^\beta_{(c)}\partial_\beta = \partial_{(c)}$, and the short notation for the covariant derivative, $\nabla_\alpha = ;\alpha$. Cylindrical coordinates, corresponding tetrad, Ricci rotation coefficients and the uniform magnetic field are determined by relations

$$x^\alpha = (t, r, \phi, z), \quad dS^2 = dt^2 - dr^2 - r^2 d\phi^2 - dz^2,$$

$$g_{\alpha\beta} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix}, g_{\alpha\beta} = \eta^{ab}e_{(a)\alpha}e_{(b)\beta}, \quad e^\alpha_{(a)} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{r} & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix},$$

$$\gamma_{122} = \frac{1}{r}, \quad \gamma_{122} = -\gamma_{212} = +\gamma_{21}{}^2 = e^\alpha_{(1);\alpha} = \frac{1}{r},$$

$$A_\phi = -Br^2/2, F_{12} = e^r_{(1)} e^\phi_{(2)} F_{r\phi} = -B \quad (F_{12} = B_3 = -B).$$

We readily find an explicit form of the above equations for the case under consideration

$$\partial_0 \Phi_0 - (\partial_r + \frac{1}{r})\Phi_1 - \frac{1}{r}\hat{\partial}_\phi \Phi_2 - \partial_z \Phi_3 - iM\Phi = 0,$$

$$\begin{vmatrix} \partial_0 \Phi \\ \partial_r \Phi \\ \frac{1}{r}\hat{\partial}_\phi \Phi \\ \partial_z \Phi \end{vmatrix} + ie\gamma B \begin{vmatrix} 0 \\ -\Phi_2 \\ +\Phi_1 \\ 0 \end{vmatrix} + ie\sigma B \begin{vmatrix} 0 \\ -\frac{1}{r}\hat{\partial}_\phi \Phi \\ +\partial_r \Phi \\ 0 \end{vmatrix} - iM \begin{vmatrix} \Phi_0 \\ \Phi_1 \\ \Phi_2 \\ \Phi_3 \end{vmatrix} = 0,$$

where $\hat{\partial}_\phi = \partial_\phi + ieBr^2/2$. Let us apply the substitution (they correspond to diagonalization of operators of energy, third projection of the total angular momentum and the third projection of the linear momentum)

$$\Psi = \begin{vmatrix} H_1 \\ H_2 \end{vmatrix}, \quad H_1 = e^{-i\epsilon t} e^{im\phi} e^{ikz} f(r), H_2 = e^{-i\epsilon t} e^{im\phi} e^{ikz} \begin{vmatrix} f_0(r) \\ f_1(r) \\ f_2(r) \\ f_3(r) \end{vmatrix};$$

taking in mind the notation $i\frac{m + eBr^2/2}{r} = i\mu(r)$ we get 5 equations

$$\epsilon f + Mf_0 = 0, \quad kf - Mf_3 = 0, \quad -i\epsilon f_0 - (\frac{d}{dr} + \frac{1}{r}) f_1 - \frac{1}{r} i\mu f_2 - ikf_3 - iMf = 0,$$

$$(\frac{d}{dr} + e\sigma B\frac{\mu}{r}) f - ie\gamma Bf_2 - iMf_1 = 0, \quad i(\frac{\mu}{r} + e\sigma B\frac{d}{dr}) f + ie\gamma Bf_1 - iMf_2 = 0.$$

Eliminating the variables f_0, f_3: $f_0 = -\frac{\epsilon}{M} f$, $f_3 = \frac{k}{M} f$, we derive a system of 3 equation:

$$+i\epsilon \frac{\epsilon}{M} f - (\frac{d}{dr} + \frac{1}{r}) f_1 - \frac{1}{r} i\mu f_2 - ik\frac{k}{M} f - iMf = 0;$$

$$(\frac{d}{dr} + e\sigma B\frac{\mu}{r}) f - ie\gamma Bf_2 - iMf_1 = 0, \quad i(\frac{\mu}{r} + e\sigma B\frac{d}{dr}) f + ie\gamma Bf_1 - iMf_2 = 0. \qquad (1)$$

By physical reason we make change $\gamma \to i\gamma$, then we get

$$f_1 = \frac{1}{M^2 - e^2\gamma^2 B^2}[-ie\gamma B(\frac{\mu}{r} + e\sigma B\frac{d}{dr}) - iM(\frac{d}{dr} + e\sigma B\frac{\mu}{r})] f,$$

$$f_2 = \frac{1}{M^2 - e^2\gamma^2 B^2}[e\gamma B(\frac{d}{dr} + e\sigma B\frac{\mu}{r}) + M(\frac{\mu}{r} + e\sigma B\frac{d}{dr})] f.$$

We substitute this result into the first equation in (1), allowing for the explicit form of $\mu(r)$; in this way we derive a second order equation for function f,

$$\frac{d^2 f}{dr^2} + \frac{1}{r}\frac{df}{dr} + (-\frac{m^2}{r^2} - \frac{e^2 B^2 r^2}{4} + A) f = 0,$$

where

$$A = -meB + \frac{e^2 B^2 (\sigma M + \gamma)}{M + e^2\gamma\sigma B^2} + \frac{(\epsilon^2 - k^2 - M^2)(-e^2\gamma^2 B^2 + M^2)}{M(M + e^2\gamma\sigma B^2)}.$$

In the variable $x = eBr^2/2$, this equation reads

$$\frac{d^2 f}{dx^2} + \frac{1}{x}\frac{df}{dx} + (-\frac{1}{4}\frac{m^2}{x^2} - \frac{1}{4} + \frac{A}{2eBx}) f = 0;$$

its solution is searched in the form $f = x^a e^{cx} F(x)$:

$$x\frac{d^2F}{dx^2} + \left(2a+1+2cx\right)\frac{dF}{dx} + [(c^2 - \frac{1}{4})x + \frac{4a^2 - m^2}{4x} + 2ac + c + \frac{A}{2eB}]F = 0.$$

Let $a = |m/2|, c = -1/2$, then the equation simplifies

$$x\frac{d^2F}{dx^2} + \left(2a+1-x\right)\frac{dF}{dx} + (-a - \frac{1}{2} + \frac{A}{2eB})F = 0$$

and it is confluent hypergeometric equations

$$x\frac{d^2F}{dx^2} + (\beta - x)\frac{dF}{dx} - \alpha F = 0, \quad \beta = 2a+1, \quad \alpha = a + \frac{1}{2} - \frac{A}{2eB}.$$

Imposing the usual polynomial requirement $\alpha = -n$,

$$a + \frac{1}{2} - \frac{A}{2eB} = -n \quad \Rightarrow \quad \frac{|m|}{2} + \frac{1}{2} - \frac{A}{2eB} = -n, n = 0,1,2,...;$$

and allowing for explicit form of A, we obtain formula for energies

$$\epsilon^2 = M^2 + k^2 + \frac{MeB\left(N-1/2\right)\left(1+eB\sigma\right)}{(M - eB\gamma)} + \frac{MeB\left(N+1/2\right)\left(1-eB\sigma\right)}{(M + eB\gamma)}.$$

where $N = (2n + |m| + m + 1)/2, n = 0,1,2,...$; parameter N takes half-integer values: $N = 1/2, 3/2, ...$. This relation may be re-written differently

$$\epsilon^2 - M^2 - k^2 = (1 - \frac{\gamma^2 B^2}{M^2})^{-1}[\frac{2BN}{M^2}(1 + \sigma B\frac{\gamma B}{M}) - B(\sigma B + \frac{\gamma B}{M})].$$

In this presentation, we can readily describe conditions when the influence of the additional characteristics γ and σ will be small:

$$B\sigma << 1 \Rightarrow \sigma << \frac{1}{B}; \qquad \frac{B\gamma}{M} << 1.$$

Let us transform the last formulas to dimensionless quantities:

$$E = \frac{\epsilon}{M}, \quad K = \frac{k}{M}, \quad x = \frac{X}{M^2}, \quad b = \frac{B}{M^2},$$

$$M\gamma = \Gamma, \quad M^2\sigma = \Sigma, \qquad \frac{B\gamma}{M} = b\Gamma, \quad B\sigma = b\Sigma.$$

The influence of additional characteristic is small if two inequality are satisfy

$$\Gamma << \frac{1}{b}, \quad \Sigma << \frac{1}{b};$$

and the formula for energy levels takes on the form (recall that $x = 2bN$)

$$E = \sqrt{1 + K^2 + \frac{1}{1 - b^2\nu^2}[x(1 + b\Sigma \cdot b\Gamma) - b(b\Sigma + b\Gamma)]}.$$

Let us study this spectrum numerically for several sets of parameters:

$$\Sigma = 0, \Gamma = 0, \quad K = 0.1, b = 0.2, \quad N = \frac{1}{2}, \frac{3}{2}, ..., \frac{15}{2},$$

$$1.1, 1.268, 1.417, 1.552, 1.676, 1.791, 1.9, 2.0025;$$

$$\Sigma = 0.02, \Gamma = 0.02, \quad K = 0.1, b = 0.2, \quad N = \frac{1}{2}, \frac{3}{2}, ..., \frac{15}{2},$$

$$1.099, 1.268, 1.417, 1.551, 1.675, 1.791, 1.899, 2.002;$$

$$\Sigma = 0.02, \Gamma = 0.02, \quad K = 0.01, b = 0.2, \quad N = \frac{1}{2}, \frac{3}{2}, ..., \frac{15}{2},$$

$$1.094, 1.264, 1.413, 1.548, 1.672, 1.788, 1.897, 1.999;$$

$$\Sigma = 0, \Gamma = 0.02, \quad K = 0.01, b = 0.2, \quad N = \frac{1}{2}, \frac{3}{2}, ..., \frac{15}{2},$$

$$1.0951, 1.264, 1.414, 1.549, 1.673, 1.788, 1.897, 1.999;$$

$$\Sigma = 0.02, \Gamma = 0, \quad K = 0.01, b = 0.2, \quad N = \frac{1}{2}, \frac{3}{2}, ..., \frac{15}{2},$$

$$1.095, 1.264, 1.414, 1.549, 1.673, 1.788, 1.897, 1.999.$$

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ЧУВСТВИТЕЛЬНОСТЬ ПОЛЯРИЗАЦИОННЫХ ФРАКЦИЙ W-БОЗОНОВ К CP-ЧЁТНЫМ АНОМАЛЬНЫМ КОНСТАНТАМ ТРЁХБОЗОННЫХ ВЕРШИН В ПРОЦЕССЕ ПАРНОГО РОЖДЕНИЯ НА CMS LHC

В работе рассмотрены эффекты влияния CP-чётных аномальных констант трёхбозонного и $\mathrm{WW\gamma}$ и WWZ-взаимодействия на значения сечений и поляризационных фракций для процесса $\mathrm{pp} \to \mathrm{W^+W^-} + X \to \ell^+\ell^-\nu_\ell\bar{\nu}_\ell + \mathrm{X}$ с поляризованным $\mathrm{W^+}$-бозоном в рамках экспериментальных ограничений на кинематические переменные для третьего сеанса работы эксперимента CMS.

Ключевые слова: бозон, кварк, партон, протон, аномальные параметры.